

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/01, 0606/01

Paper 1

October/November 2009

MARK SCHEME
Maximum Mark: 80

IMPORTANT NOTICE

Mark Schemes have been issued on the basis of **one** copy per Assistant examiner and **two** copies per Team Leader.

1(i) $2a^3 - 7a^2 + 7a^2 + 16 = 0$ leading to $a^3 = -8$, $a = -2$ [2] (ii) $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ M1 M1 for substitution of $x = -\frac{1}{2}$ 2 (i) [2] M1 for substitution of $x = -\frac{1}{2}$ 2 (i) [3] $2a + 3$ [2] $3a + 2a + 3$ [2] [2] [3] $a + 3a +$		1	
(ii) $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ M1 M1 for substitution of $x = -\frac{1}{2}$ 2 (i) (ii) (ii) [2] B1.B1 correct order $ \begin{bmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 3 & 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 1 & 2 \\ 3 & 3 & 2 & 4 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 2 & 5 & 5 & 0 & 2 & 3 \\ 2 & 2 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4(2k+1)^2 = 4(k+2) & 4k^2 + 3k - 1 = 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ B1.B1 correct order $ \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ B1.B1 correct order $ \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ B1.B1 pro each matrix, must be in correct order $ \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ B1.B1 pro each matrix, must be in correct order $ \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ B1.B1 pro each matrix, must be in correct order $ \begin{bmatrix} 7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ M1 for use of $b^2 - 4ac$, Correct quadratic equation $ \begin{bmatrix} 8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & (13 - 3y)^2 + 3y^2 = 43 & 1 & 1 & 1 \end{bmatrix} $ M1 for correct attempt at solution A1 for correct quadratic $ \begin{bmatrix} 7 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ M1 for correct attempt at solving quadratic $ \begin{bmatrix} 9 & 3 & 1 & 7 & 2 & 1 & 1 & 1 & 1 \\ 4 & (13 - 3y)^2 + 3y^2 = 43 & 1 & 1 & 1 \end{bmatrix} $ A1 for each correct pair $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ A1 for each correct quadratic $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ A1 for each correct quadratic $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ A1 for each error $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ A1 for correct attempt at solving quadratic $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} $ A1 for each error $ \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 $			M1 for use of $x = a$
(ii) $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ M1 M1 for substitution of $x = -\frac{1}{2}$ 2 (i) (ii) $\begin{cases} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{cases}$ (ii) $\begin{cases} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{cases}$ (ii) $\begin{cases} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{cases}$ (b) $\begin{cases} 43 \\ 32 \\ 22 \end{cases}$ B1, B1 for each matrix, must be in correct order 3 $4(2k+1)^2 = 4(k+2)$ M1 for use of ' $b^2 - 4ac$ ' Correct quadratic equation 4 $(13-3y)^2 + 3y^2 = 43$ M1 for both 1 values 4 $(13-3y)^2 + 3y^2 = 43$ M1 M1 for eliminating one variable 6 $(2y^2 - 13y + 21) = 0$ (or $2(2x^2 - 13x + 20) = 0$) 6 $(2y - 7)(y - 3) = 0$ (or $(2x - 5)(x - 4) = 0$ M1 M1 for correct attempt at solving quadratic 9 $= 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ A1, A1 A1 for each correct pair 1 (or $x = 4$ or $\frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$) [5] 5 (i) $(3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ AC $= \sqrt{22}$ M1 M1 for use of Pythagoras A1 M1 for correct ratio	leading to $a^3 = -8$, $a = -2$		
$2\left(-\frac{1}{2}\right)^{3} - 7\left(-\frac{1}{2}\right)^{2} - 14\left(-\frac{1}{2}\right) + 16$ $= 21$ $2 \text{ (i)} \qquad \text{ (ii)} \qquad \qquad$	(ii)		
		M1	M1 for substitution of $x = -\frac{1}{2}$
2 (i) (ii) (ii) $\begin{bmatrix} 2 \\ 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{bmatrix} \begin{pmatrix} 43 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{bmatrix} 43 \\ 32 \\ 35 \\ 22 \end{bmatrix}$ B1,B1 B1 for each matrix, must be in correct order $\begin{bmatrix} 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{bmatrix} \begin{pmatrix} 43 \\ 3 \\ 2 \\ 35 \\ 22 \end{pmatrix}$ B2,1,0 -1 for each error $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ 3 $4(2k+1)^2 = 4(k+2)$ $4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}$,-1 M1 M1 for use of ' $b^2 - 4ac$ ' Correct quadratic equation M1 M1 for correct attempt at solution A1 for both 1 values 4 $(13-3y)^2 + 3y^2 = 43$ $(or \ x^2 + \frac{(13-x)^2}{3} = 43)$ $6(2y^2 - 13y + 21) = 0$ $(or \ 2(2x^2 - 13x + 20) = 0)$ $(2y - 7)(y - 3) = 0$ $(or \ (2x - 5)(x - 4) = 0$ $y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ $(or \ x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$ $(or \ x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$ $5 (i) \ (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ $AC = \sqrt{22}$ $AC = \sqrt{22}$ M1 M1 for use of Pythagoras A1 M1 for correct ratio M1 M1 for correct ratio M1 M1 for correct ratio M1 M1 for rationalising	$2\left(-\frac{1}{2}\right)^{-1}\left(-\frac{1}{2}\right)^{-14}\left(-\frac{1}{2}\right)^{+16}$	IVII	2
2 (i) (ii) $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 3 & 4(2k+1)^2 = 4(k+2) \\ 4k^2 + 3k - 1 = 0 \\ 12 \end{pmatrix}$ $\begin{pmatrix} 4 & (13-3y)^2 + 3y^2 = 43 \\ (or & x^2 + \frac{(13-x)^2}{3} = 43) \\ 6(2y^2 - 13y + 21) = 0 \\ (or & (2(2x^2 - 13x + 20) = 0) \\ (or & (2x - 5)(x - 4) = 0 \\ y = 3 \text{ or } \frac{7}{2} \begin{pmatrix} x = \frac{5}{2} \text{ or } 4 \\ AC = \sqrt{22} \end{pmatrix}$ $\begin{pmatrix} 4 & (13 - 3y)^2 + 3y^2 = 43 \\ A1 & A1 \text{ for correct attempt at solution A1 for each correct quadratic} \\ A1 & A1 \text{ for correct quadratic} \\ A1 & A1 \text{ for correct attempt at solving quadratic} \\ A1 & A1 \text{ for correct attempt at solving quadratic} \\ A1 & A1 \text{ for each correct pair} \\ A1 & A1 \text{ for each correct pair} \\ A1 & A1 \text{ for each correct pair} \\ A2 & A2 & A3 & A4 \text{ for or each error} \\ A4 & A4 \text{ for correct attempt at solving quadratic} \\ A4 & A4 \text{ for correct quadratic} \\ A4 & A4 \text{ for correct attempt at solving quadratic} \\ A4 & A4 \text{ for each correct pair} \\ A4 & A4 \text{ for each correct pair} \\ A4 & A4 \text{ for each correct pair} \\ A5 & A7 & A7 & A7 & A1 \text{ for each correct pair} \\ A6 & A7 & A7 & A1 \text{ for or each correct pair} \\ A7 & A1 & A1 \text{ for or each correct pair} \\ A1 & A1 \text{ for each correct pair} \\ A2 & A3 & A4 & A4 \text{ for each correct pair} \\ A4 & A4 & A4 \text{ for each correct pair} \\ A5 & A7 & A7 & A1 \text{ for or each error} \\ A4 & A4 & A4 \text{ for correct attempt at solving quadratic} \\ A4 & A4 & A4 & A4 \text{ for each correct pair} \\ A5 & A7 & A7 & A7 & A1 \text{ for each correct pair} \\ A4 & A4 & A4 & A4 & A4 \text{ for each correct pair} \\ A4 & A4 & A4 & A4 & A4 & A4 & A4 \text{ for each correct pair} \\ A5 & A7 & A7 & A4 & A4 & A4 & A4 & A4 & A4$	= 21	A1	
$\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \\ 35 \\ 22 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 35 \\ 22 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 32 \\ 35 \\ 35 \\ 35 \\ 35 \\ 35 \\ 35 \\ 35$	2 (1)		D1 for a selection was the in
[2] $ 3$			·
[2] $ 3$	$\begin{bmatrix} 0 & 3 & 1 & 2 & 3 & 1 & 3 \\ 3 & 2 & 4 & 3 & 3 & 3 & 3 \end{bmatrix}$	[-]	0.522.000
[2] $ 3$	$\begin{vmatrix} 3 & 2 & 1 & 3 \\ 2 & 5 & 5 & 0 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 35 \end{vmatrix}$	D2 1 0	1.6
[2] $ 3$		B2,1,0	-1 for each error
Al Correct quadratic equation $4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}$, -1 $4 (13 - 3y)^2 + 3y^2 = 43$ $(or x^2 + \frac{(13 - x)^2}{3} = 43) 6(2y^2 - 13y + 21) = 0 (or 2(2x^2 - 13x + 20) = 0) (2y - 7)(y - 3) = 0 (or (2x - 5)(x - 4) = 0 y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right) (or x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right) 5 (i) (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22 AC = \sqrt{22} M1 M1 \text{ for correct attempt at solving quadratic} A1,A1 A1 \text{ for each correct pair} A1,A1 A1 \text{ for each correct pair} [2] M1 M1 \text{ for use of Pythagoras} A1 [2] M1 M1 \text{ for correct ratio} M1 M1 for or o$		[2]	
$4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}$, -1 $4 (13 - 3y)^2 + 3y^2 = 43$ $(or x^2 + \frac{(13 - x)^2}{3} = 43) 6(2y^2 - 13y + 21) = 0 (or 2(2x^2 - 13x + 20) = 0) (2y - 7)(y - 3) = 0 (or (2x - 5)(x - 4) = 0 y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right) (or x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right) 5 (i) (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22 AC = \sqrt{22} (ii) \tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}} M1 M1 \text{ for correct attempt at solving quadratic} A1, A1 A1 \text{ for each correct pair} A1 A1 \text{ for each correct pair} M1 M1 \text{ for use of Pythagoras} A1 [2] M1 M1 \text{ for correct ratio} M1 M1 \text{ for rationalising}$	$3 4(2k+1)^2 = 4(k+2)$		M1 for use of ' $b^2 - 4ac$ '
leading to $k = \frac{1}{4}$, -1 M1 A1 M1 for correct attempt at solution A1 for both 1 values M1 A1 for both 1 values M1 M1 for eliminating one variable M1 A1 for correct quadratic M1 A1 for correct quadratic A1 for correct quadratic A1 for correct attempt at solving quadratic M1 M1 for correct quadratic A1 for correct attempt at solving quadratic M1 A1 for correct attempt at solving quadratic A1,A1 A1 for each correct pair The following quadratic A1,A1 A1 for each correct pair Solving quadratic A1,A1 A1 for each correct pair M1 M1 for use of Pythagoras A1 [2] M1 M1 for correct ratio		A1	Correct quadratic equation
4 $(13-3y)^2 + 3y^2 = 43$ M1 M1 for eliminating one variable $(cor x^2 + \frac{(13-x)^2}{3} = 43)$ A1 for correct quadratic $(cor 2(2x^2 - 13x + 20) = 0)$ A1 A1 for correct quadratic $(cor 2(2x^2 - 13x + 20) = 0)$ M1 M1 for correct attempt at solving quadratic $(cor 2(2x - 5)(x - 4) = 0)$ A1,A1 A1 for each correct pair $(cor x = 4 \text{ or } \frac{5}{2} \text{ or } 4)$ A1 for use of Pythagoras A1 [2] M1 M1 for correct ratio $(cor x = 4 \text{ or } \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for correct ratio M1, A1 M1 for rationalising		M1	M1 for correct attempt at solution
4 $(13-3y)^2 + 3y^2 = 43$ M1 M1 for eliminating one variable $(or x^2 + \frac{(13-x)^2}{3} = 43)$ A1 A1 for correct quadratic $(or 2(2x^2-13x+20)=0)$ A1 M1 for correct attempt at solving quadratic $y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ A1,A1 A1 for each correct pair $(or x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right))$ [5] 5 (i) $(3+\sqrt{2})^2 + (3-\sqrt{2})^2 = 22$ AC AC $= \sqrt{22}$ M1 M1 for use of Pythagoras A1 [2] M1 M1 for correct ratio $(3-\sqrt{2})(3-\sqrt{2}) = \frac{11-6\sqrt{2}}{3+\sqrt{2}}$ M1, A1 M1 for rationalising	$\frac{1}{4}$		_
$(or \ x^2 + \frac{(13 - x)^2}{3} = 43)$ $6(2y^2 - 13y + 21) = 0$ $(or \ 2(2x^2 - 13x + 20) = 0)$ $(2y - 7)(y - 3) = 0$ $(or \ (2x - 5)(x - 4) = 0$ $y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ $(or \ x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$ $5 \ (i) \ (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ $AC = \sqrt{22}$ $M1$ $M1 \text{ for correct attempt at solving quadratic}$ $A1,A1 \text{ A1 for each correct pair}$ $[5]$ $M1 \text{ M1 for use of Pythagoras}$ $A1 \text{ [2]}$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for rationalising}$		[4]	
$(\text{or } x^2 + \frac{(13 - x)^2}{3} = 43)$ $6(2y^2 - 13y + 21) = 0$ $(\text{or } 2(2x^2 - 13x + 20) = 0)$ $(2y - 7)(y - 3) = 0$ $(\text{or } (2x - 5)(x - 4) = 0$ $y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ $(\text{or } x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$ $5 \text{ (i) } (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ $AC = \sqrt{22}$ $AC = \sqrt{22}$ AI $A1 \text{ for correct quadratic}$ $A1,A1 \text{ A1 for each correct pair}$ $[5]$ $M1 \text{ M1 for use of Pythagoras}$ $A1$ $[2]$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for rationalising}$	$4 (13-3y)^2 + 3y^2 = 43$	M1	M1 for eliminating one variable
$6(2y^2 - 13y + 21) = 0$ $(or 2(2x^2 - 13x + 20) = 0)$ $(2y - 7)(y - 3) = 0$ $(or (2x - 5)(x - 4) = 0$ $y = 3 \text{ or } \frac{7}{2} \left(x = \frac{5}{2} \text{ or } 4\right)$ $(or x = 4 \text{ or } \frac{5}{2} \left(y = \frac{7}{2} \text{ or } 3\right)$ $A1 $			
$(or 2(2x^2-13x+20)=0)$ $(2y-7)(y-3)=0$ $(or (2x-5)(x-4)=0$ $y=3 \text{ or } \frac{7}{2} \left(x=\frac{5}{2} \text{ or } 4\right)$ $(or x=4 \text{ or } \frac{5}{2} \left(y=\frac{7}{2} \text{ or } 3\right)$ $A1,A1$ $A1 \text{ for each correct pair}$ $(or x=4 \text{ or } \frac{5}{2} \left(y=\frac{7}{2} \text{ or } 3\right)$ $A2 = \sqrt{22}$ $A2 = \sqrt{22}$ $A3 = \sqrt{2}$ $A1 = \sqrt{2}$ $A1 = \sqrt{2}$ $A1 = \sqrt{2}$ $A2 = \sqrt{2}$ $A1 = \sqrt{2}$ $A2 = \sqrt{2}$ $A2 = \sqrt{2}$ $A3 = \sqrt{2}$	$(\text{or } x^2 + \frac{(13-x)}{3} = 43)$		
(or $2(2x^2 - 13x + 20) = 0$) (2y - 7)(y - 3) = 0 (or $(2x - 5)(x - 4) = 0$ $y = 3$ or $\frac{7}{2}$ $\left(x = \frac{5}{2} \text{ or } 4\right)$ (or $x = 4$ or $\frac{5}{2}$ $\left(y = \frac{7}{2} \text{ or } 3\right)$) (or $x = 4$ or $\frac{5}{2}$ $\left(y = \frac{7}{2} \text{ or } 3\right)$) (ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for use of Pythagoras A1 [2] M1 M1 for correct ratio M1 for correct ratio M1 for correct ratio M1 for correct ratio	$6(2y^2 - 13y + 21) = 0$	Δ1	A1 for correct quadratic
$(\text{or } (2x-5)(x-4)=0$ $y=3 \text{ or } \frac{7}{2} \left(x=\frac{5}{2} \text{ or } 4\right)$ $(\text{or } x=4 \text{ or } \frac{5}{2} \left(y=\frac{7}{2} \text{ or } 3\right))$ $5 \text{ (i) } \left(3+\sqrt{2}\right)^2+\left(3-\sqrt{2}\right)^2=22$ $AC=\sqrt{22}$ $(\text{ii) } \tan A=\frac{3-\sqrt{2}}{3+\sqrt{2}}$ $\left(\frac{3-\sqrt{2}}{3+\sqrt{2}}\right)^3-\frac{11-6\sqrt{2}}{7}$ $M1$ $M1 \text{ for correct ratio}$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for rationalising}$	(or $2(2x^2-13x+20)=0$)	Ai	At for correct quadratic
$(\text{or } (2x-5)(x-4)=0$ $y=3 \text{ or } \frac{7}{2} \left(x=\frac{5}{2} \text{ or } 4\right)$ $(\text{or } x=4 \text{ or } \frac{5}{2} \left(y=\frac{7}{2} \text{ or } 3\right))$ $5 \text{ (i) } \left(3+\sqrt{2}\right)^2+\left(3-\sqrt{2}\right)^2=22$ $AC=\sqrt{22}$ $(\text{ii) } \tan A=\frac{3-\sqrt{2}}{3+\sqrt{2}}$ $\left(\frac{3-\sqrt{2}}{3+\sqrt{2}}\right)^3-\frac{11-6\sqrt{2}}{7}$ $M1$ $M1 \text{ for correct ratio}$ $M1 \text{ M1 for correct ratio}$ $M1 \text{ M1 for rationalising}$	(2y-7)(y-3)=0	M1	M1 for correct attempt at colving
(or $x = 4$ or $\frac{5}{2}$ ($y = \frac{7}{2}$ or 3)) [5] 5 (i) $(3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ AC = $\sqrt{22}$ (ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for use of Pythagoras [2] M1 M1 for correct ratio $\frac{(3 - \sqrt{2})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{11 - 6\sqrt{2}}{7}$ M1, A1 M1 for rationalising		IVII	
(or $x = 4$ or $\frac{5}{2}$ ($y = \frac{7}{2}$ or 3)) [5] 5 (i) $(3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 = 22$ AC = $\sqrt{22}$ (ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for use of Pythagoras [2] M1 M1 for correct ratio $\frac{(3 - \sqrt{2})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{11 - 6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	$y = 3 \text{ or } \frac{7}{7} \left(x = \frac{5}{2} \text{ or } 4 \right)$	Δ1 Δ1	A1 for each correct pair
5 (i) $(3+\sqrt{2})^2 + (3-\sqrt{2})^2 = 22$ $AC = \sqrt{22}$ A1 $[2]$ (ii) $\tan A = \frac{3-\sqrt{2}}{3+\sqrt{2}}$ $\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$ M1 M1 for use of Pythagoras $M1$ M1 for correct ratio $M1$ M1 for rationalising	_ (_ /	111,111	Til for each correct pair
AC = $\sqrt{22}$ (ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for correct ratio $\frac{(3 - \sqrt{2})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{11 - 6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	(or $x = 4$ or $\frac{5}{2}$ ($y = \frac{7}{2}$ or 3))	[5]	
AC = $\sqrt{22}$ (ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for correct ratio $\frac{(3 - \sqrt{2})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{11 - 6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	5 (i) $(3+\sqrt{2})^2 + (3-\sqrt{2})^2 = 22$	M1	M1 for use of Pythagoras
(ii) $\tan A = \frac{3 - \sqrt{2}}{3 + \sqrt{2}}$ M1 M1 for correct ratio $\frac{(3 - \sqrt{2})(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{11 - 6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	/ _ /	A1	
$\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	_		
$\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$ M1, A1 M1 for rationalising	(ii) $\tan A = \frac{3 - \sqrt{2}}{\sqrt{2}}$	N/1	M1 for compet with
	$3+\sqrt{2}$	IVII	IVIT TOF COFFECT PATIO
	$(3-\sqrt{2})(3-\sqrt{2})$ $11-6\sqrt{2}$	3.61	
	$\left \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} \right = \frac{11-6\sqrt{2}}{7}$	M1, A1	M1 for rationalising
	,	[3]	

6 (i) $3x^2 - 10x - 8 = 0$	M1	M1 for attempt to solve quadratic
(3x+2)(x-4)=0		T
, , , ,		
critical values $-\frac{2}{3}$,4	A1	A1 for critical values
3		
$A = \{x : -\frac{2}{3} \le x \le 4\}$	A1	
(ii)	[3]	
$B = \{x : x \ge 3\}$	B1	B1 for values of <i>x</i> that define <i>B</i> .
$A \cap B = \{x : 3 \le x \le 4\}$	B1	B1 for attempt to combine the sets
,	[2]	correctly and correct use of notation
7 (i) $^{13}C_8 = 1287$	M1, A1	M1 for correct C notation
	[2]	
(ii) 7 teachers, 1 student: 6	B1	
6 teachers, 2 students ${}^{7}C_{6} \times {}^{6}C_{2}$:105	B1 B1	
5 teachers, 3 students ${}^{7}C_{5} \times {}^{6}C_{3}$:420	DI	
531	B1	
331	[4]	
8 (i) When $t = 0$, $N = 1000$	B1	
	[1]	
(ii) $\frac{\mathrm{d}N}{\mathrm{d}t} = -1000k\mathrm{e}^{-kt}$		
dt	M1	M1 for differentiation
when $t = 0$, $\frac{dN}{dt} = -20$ leading to	B1	AN
	DI	B1 for $\frac{dN}{dt} = -20$ stated
$k = \frac{1}{50}$	A1	d <i>t</i>
	[3]	
(iii) $500 = 1000e^{-kt}$	M1	M1 for attempt to formulate
		equation using half life
$t = -50 \ln \frac{1}{2}$ leading to 34.7 mins	M1	M1 for a correct attempt at solution
2 reading to 5 m mins	A1	_
10	[3]	, ,10
9 (i) $20 \times -2(1-2x)^{19}$	B1,B1	B1 for 20 and $(1-2x)^{19}$
	[2]	B1 for -2
	[2]	
(ii) $x^2 \frac{1}{x} + 2x \ln x$	M1	M1 for attempt to differentiate a
x	B1	product.
	A1	B1 for $\frac{1}{-}$
	507	X
	[3]	
(iii)	M1	M1 for attempt to differentiate a
	B1	quotient.
$\frac{x(2\sec^2(2x+1))-\tan(2x+1)}{x^2}$	A1	B1 for differentiation of $tan(2x+1)$
χ^{-}	[3]	

10 (i) $\frac{dy}{dx} = 9x^2 - 4x + 2$ at P grad = 7 tangent $y - 3 = 7(x - 1)$	M1 A1 M1 A1 [4]	M1 for differentiation and attempt to find gradient M1 for attempt to find tangent equation, allow unsimplified
(ii) at Q , $7x-4=3x^3-2x^2+2x$ leading to $3x^3-2x^2-5x+4=0$ $(x-1)(3x^2+x-4)-0$ (x-1)(3x+4)(x-1)=0 leading to $x=-\frac{4}{3}$, $y=-\frac{40}{3}$	M1 B1,M1 M1 A1 [5]	M1 for equating tangent and curve equations B1 for realising (x - 1) is a factor and attempt to factorise M1 for factorisation and attempt to solve quadratic A1 for both
11 (a) $\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$ $= \frac{1}{\cos \theta \sin \theta}$	M1 M1	M1 for attempt to get in terms of sin and cos and attempt to get one fraction M1 for use of identity
$= \cos \cot \theta \sec \theta$ (b)(i) $\tan x = 3\sin x$ $\frac{\sin x}{\cos x} = 3\sin x$ $\sin x - 3\sin x \cos x = 0$	A1 [3] M1	M1 for use of $\tan x = \frac{\sin x}{\cos x}$ and correct attempt to solve
leading to $\cos x = \frac{1}{3}$, $\sin x = 0$ $x = 70.5^{\circ}, 289.5^{\circ}$ and $x = 180^{\circ}$	A1√A1 B1	$\sqrt{A1}$ on their <i>x</i> = 70.5° B1 for <i>x</i> = 180°
(ii) $2\cot^2 y + 3\csc y = 0$ $2(\csc^2 y - 1) + 3\csc y = 0$	[4] M1	M1 for use of correct identity
$2\cos e^{2}y + 3\cos exy - 2 = 0$ $(2\cos exy - 1)(\cos exy + 2) = 0$ leading to $\sin y = -\frac{1}{2}$, $y = \frac{7\pi}{6}$, $\frac{11\pi}{6}$	M1 M1 A1,A1 [5]	M1 for attempt to solve quadratic M1 for dealing with cosec

12 EITHER		
(i) $\pi r^2 h = 1000$, leading to	M1	M1 for attempt to use volume
_		
$h = \frac{1000}{\pi r^2}$	A1	
	[2]	
(ii) $A = 2\pi rh + 2\pi r^2$	M1	M1 for attempt to use surfece erec
leading to given answer	A1	M1for attempt to use surface area GIVEN ANSWER
$A = 2\pi r^2 + \frac{2000}{r}$	[2]	GIVEN / HAG WER
r = 2m + r	[-]	
(iii) $\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2}$	M1	M1 for attempt to differentiate and
$\frac{dr}{dr} = 4\pi i r^2$	A1	set to 0
when $\frac{dA}{dr} = 0$, $4\pi r = \frac{2000}{r^2}$	DM1	DM1 for solution
leading to $r = 5.42$	A1	
5 · · · · <u>-</u>	[4]	
(iv) $\frac{d^2A}{dr^2} = 4\pi + \frac{4000}{r^3}$	M1	M1 for good darivative method -
(iv) $\frac{dr^2}{dr^2} = 4\pi + \frac{r^3}{r^3}$	IVII	M1 for second derivative method or gradient method'
		gradient method
+ ve when $r = 5.42$ so min value	A1	A1 for minimum, can be given if r
		incorrect but + ve
A_{\min} =554	A1	
	[3]	
12 OR (i) $y = x + \cos 2x$	M1	M1 for attempt to differentiate
$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2\sin 2x$	A1	
	AI	
when $\frac{dy}{dx} = 0$, $\sin 2x = \frac{1}{2}$	M1	M1 for setting to 0 and attempt to
		solve
leading to $x = \frac{\pi}{12}, \frac{5\pi}{12}$	M1	M1 for correct order of operations
12 12	A1,A1	
(ii) Area = $\int_{12}^{\frac{5\pi}{12}} x + \cos 2x dx$	[6]	
(ii) Area = $\int x + \cos 2x dx$	M1	M1 for ottomat to integrate
$\frac{\pi}{12}$	M1	M1 for attempt to integrate
5-		
$= \left[\frac{x^2}{2} + \frac{1}{2}\sin 2x\right]_{\frac{\pi}{12}}^{\frac{3\pi}{12}}$		
$-\frac{1}{2} + \frac{1}{2} \sin 2x \Big _{\pi}$	A1,A1	Alfor each term correct
12	DM1	DM1 for correct use of limits
$=\frac{\pi^2}{}$		(Trig terms cancel out)
12	A1	
	[5]	